Simple Harmonic Oscillator

Equipment:

- No special safety equipment is required for this lab.
- Computer with PASCO 850 Universal Interface and PASCO Capstone
- PASCO Motion Sensor
- Vertical Stand with horizontal cross bar.
- Spring kit.
- Mass hanger.
- Set of masses.

Introduction

Imagine a spring that is suspended from a support. When no mass is attached at the end of the spring, it has a length L (called its rest length). If a mass is added to the spring, its length increases by ΔL. The equilibrium position of the mass is now a distance $L + \Delta L$ from the spring's support

What happens if the mass is pulled down a small distance ***A*** from the equilibrium position? The spring exerts a restoring force, $F = -kx$, where ***x*** is the distance the spring is pulled down and ***k*** is the force constant of the spring. The negative sign indicates that the force points opposite of the direction of the displacement of the mass. The restoring force causes the mass to oscillate or move up and down within a range of ***A*** from the equilibrium. The distance A or maximum displacement from the equilibrium is called ***an amplitude*** of oscillation. The period of oscillation for simple harmonic motion depends on the mass and the force constant of the spring.

We expect that the frequency of the oscillations will be found from:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Here, f is the frequency in hertz (Hz), k is the spring constant in N/m, and m is the mass attached to the spring, in kg.

Another measure of the oscillations is the period. This is the time for one oscillation. It is:

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

Note that the amplitude of oscillation is not present in the formulas above and, therefore, it has no effect on the period and frequency of oscillation.

Objective

- *To verify the dependence of a period of a spring-mass system acting as a simple harmonic oscillator on mass, spring constant, and amplitude.*

Part #1. Measuring the Spring-Mass System

1. Suspend a green spring from a horizontal support rod and add enough mass to the other end to stretch the spring so the coils do not touch.
2. Attach the motion sensor to the PASCO interface as appropriate.

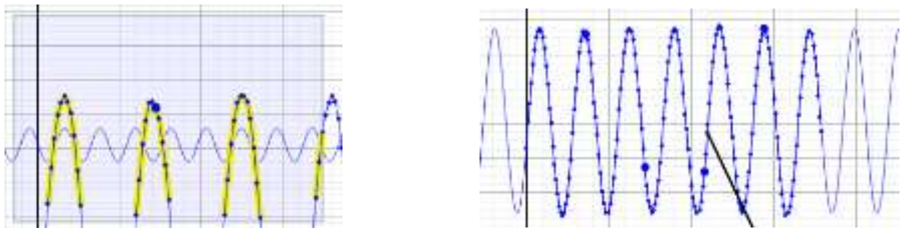
Figure 1. The PASCO 850 Interface and the two types of motion sensors: the Blue motion sensor (plug into PasPort 1) and the Black motion sensor (plug into Digital Inputs 1 & 2)

3. Open "Simple Oscillator Setup" file located online with the lab instructions. Choose the Tab corresponding to the Position vs. Time graph for your motion sensor (blue or black).
4. Place the Motion Sensor on the table with the metal disc and green LED facing up, directly under the hanging mass.
5. If the mass is not oscillating up and down, lightly tap the mass. Let it oscillate a few times so the mass hanger will move up-and-down without much side-to-side motion. Make sure the coils don't touch when the mass is at its highest point. If the coils touch, try to create a more gentle oscillation. If there is a lot of side-to-side motion, settle it down and start again.
6. While the mass is oscillating, press Record to monitor the position of the mass relative to the sensor over a period of several oscillations. If there are frequent bad data points, check the alignment of the sensor under the mass, and maybe try changing the selector switch on the sensor.
7. Rescale the data to fit the Graph window if necessary.
8. Using the Coordinate Tool, measure the time of five consecutive peaks.
9. Use the measured time to calculate the experimental value of the period of oscillations by calculating the difference between two sequential times.
10. Average the experimental periods and use it to calculate the frequency of oscillation.

11. To obtain the frequency from a Best Fit of the data series, fit the graph into a sine function.

    There are 3 relevant tools above the Capstone graph 

    a. The first is a highlighter used if necessary, to exclude some data points.
    b. The second turns the fit line on and off.
    c. The third is the little pulldown next to the fit button. Use it to select the sine function.
12. When doing a fit, make sure the fit function actually passes through the good data points! Sometimes it may be necessary to use the highlighter to include only some points. Other times, the highlighter may make things worse.



(a) (b)

Figure 2. Examples of (a) a bad sinewave fit, and (b) a good sinewave fit.

13. Record the value of ω from the fit and convert it to regular frequency, *f* in Hz with $f = \frac{\omega}{2\pi}$.
14. Compare the two methods of obtaining the frequency.

Peak Number	1	2	3	4	5
Time (s)					
Period (s)					
Average Period (s)					
Frequency from Period (Hz)					
Angular Frequency from fit, ω (s^-1)					
Frequency from fit (Hz)					

Table 1. Measured and Calculated values for an oscillator build with the ______ spring and a hanging mass of _____ *kg*.

Part#2. Period vs. Spring Constant

1. Using the same suspended mass in each trial, repeat steps 5-7 from Part #1 for each spring in the spring set. For the Spring Constant, use the number on the top of the box of springs.
2. To obtain the frequency ***do not use the Coordinate Tool***, use Best Fit Tool as described in Part #1, steps 11-13.

Color of the spring	**Green**	**White**	**Yellow**	**Blue**	**Red**
Spring Constant (N/m)					
Angular Frequency, ω (s^-1)					
Frequency (Hz)					
Squared Frequency (Hz^2)					

Table 2. Oscillation frequencies for several springs, each with the same suspended mass of ___kg.

Since the frequency is proportional to the square root of the spring constant, the frequency squared should be proportional to the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$f^2 = \frac{1}{4\pi^2 m}k$$

In a graph of f^2 vs. k, the slope should be equal to $(4\pi^2 m)^{-1}$

3. Graph Squared Frequency vs. Spring Constant. Fit the plot into a linear function and obtain the equation of the trendline.
4. Calculate $(4\pi^2 m)^{-1}$ and compare it with the slope of the graph.

	$(4\pi^2 m)^{-1}$	Slope	% Difference
Value			

Table 2a. Analysis of …

Part #3. Period vs. Mass

1. Choose one of the springs (other than the white one) in the set suspended and find the frequency of oscillation as above while changing the ***suspended mass by increments of your choice***.

Suspended Mass (kg)					
Angular Frequency, ω (s^-1)					
Frequency (Hz)					
Period (s)					
Squared Period (s^2)					

Table 3. Oscillation frequencies for several masses, each hanging from the ________ spring.

Since the period is proportional to the square root of the mass, the period squared should be proportional to the mass.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T^2 = \frac{4\pi^2}{k}m$$

In a graph of T^2 vs. m, the slope should equal $4\pi^2/k$.

2. Graph Squared Period vs. Mass and verify that the slope of this graph equals $4\pi^2/k$. Analyze the results in a Table 3a, similar in style to Table 2a above.

Part #4. Frequency vs. Amplitude

How should the period of oscillation change with the change of the amplitude? Once again, perform an experiment to investigate this. Suspend the spring from the previous experiment and add a mass of your choice. Throughout this exercise, the mass and spring constant should not change. Repeat the above procedure starting with a very small amplitude and gradually increasing it. Use the Sine fit to measure the amplitude as well as the frequencies.

Amplitude (m)					
Frequency (Hz)					

Table 4. Oscillation frequency at different amplitudes for the _____spring with the mass of ____kg.

Requirements for the Report:

The report must contain a **Header** at the top (Title of Lab, Authors, and Date)

Abstract Section must contain the following in paragraph form:

- Brief Introduction that includes objectives and basic theory of the lab. Include:
    - What makes a system to become an oscillator?
    - What quantities describe an oscillation?
    - What physical properties of an oscillator affect the oscillation?
- Methodology describing broadly what was done, using what tools, and what was measured/recorded.
- Data Summary including quantities worked into sentences.
    - *Part 1*: Investigate the data of period and frequency and discuss the relationship between the two variables. Compare the two methods of obtaining frequency.
    - *Part 2*: Investigate the data of spring constant and frequency. What is the relationship between spring constant and frequency? Does the data support the theoretical relationship? Use the graph and % difference to back up your analysis.
    - *Part 3*: Investigate the data of mass and period. What is the relationship between mass and period? Does the data support the theoretical relationship? Use the graph and % difference to back up your analysis.
    - *Part 4*: Analyze the data and establish a relationship between amplitude and frequency. Is the relationship expected?
- Conclusions based on the quantitative results. Briefly summarize the results of each part of the lab from the Data Summary.
- Sources of Error and a ballpark estimate of their contribution. DO NOT use "human error". That term is too vague to be meaningful.

Data Section must contain the following:
[Each table and graph should be labeled and descriptively captioned. You can use the captions provided in the lab manual, but input the values used by your lab group.]

- 6 Tables
- 2 Graphs